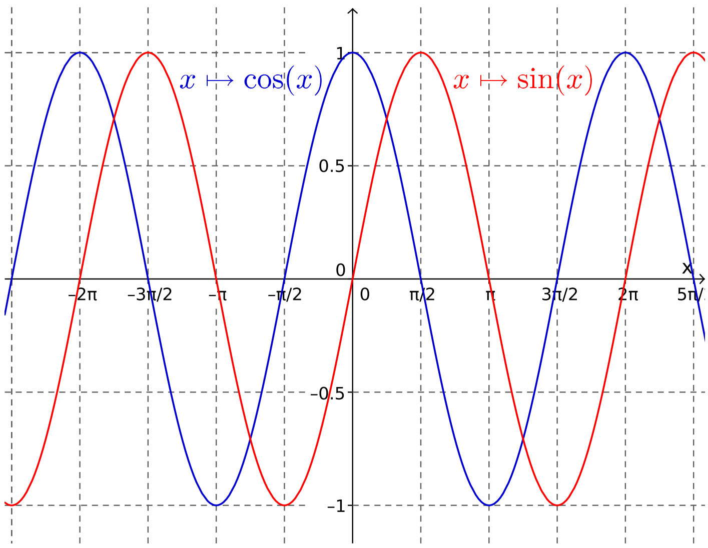
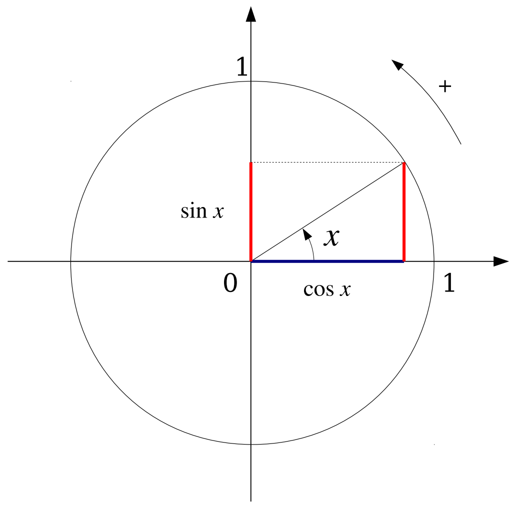
Fonctions sinusoïdales

Les fonctions *sinus* et *cosinus* sont des fonctions périodiques de période 2π.

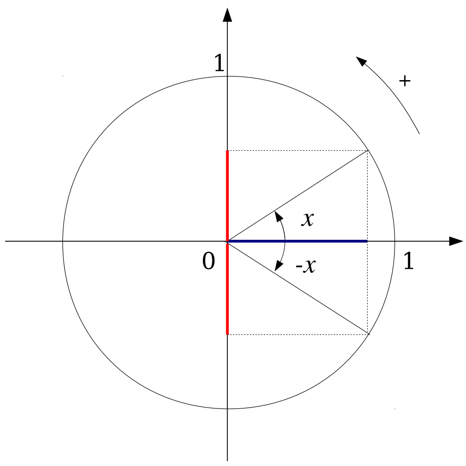




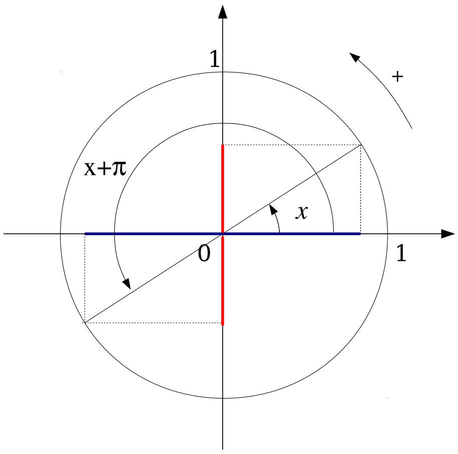
Cercle trigonométrique

$$\cos^2 x + \sin^2 x = 1$$

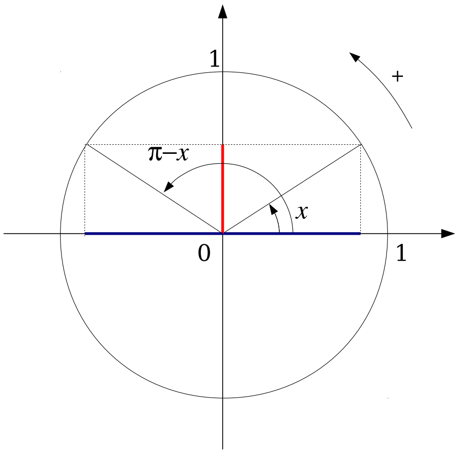
$$\cos(-x) = \cos x$$
$$\sin(-x) = -\sin x$$



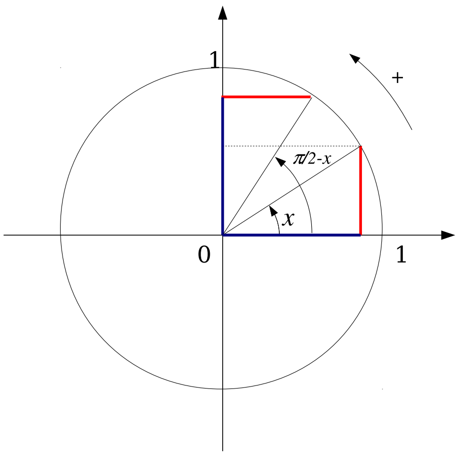
$$\cos(x+\pi) = -\cos x$$
$$\sin(x+\pi) = -\sin x$$



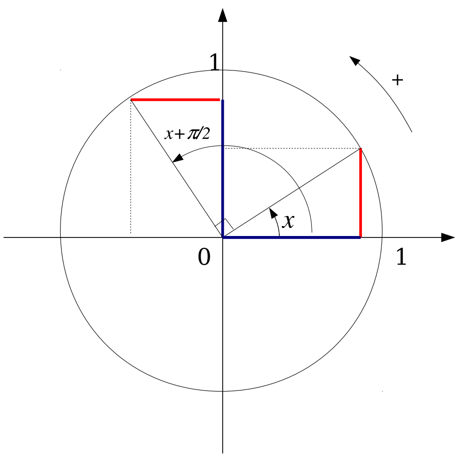
$$\cos(\pi - x) = -\cos x$$
$$\sin(\pi - x) = \sin x$$



$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$
$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$



$$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$$
$$\sin\left(x + \frac{\pi}{2}\right) = \cos x$$



x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0

Des formules suivantes :

$$\begin{cases} \cos(a+b) = \cos a \cos b - \sin a \sin b \\ \cos(a-b) = \cos a \cos b + \sin a \sin b \end{cases}$$

$$\begin{cases} \sin(a+b) = \sin a \cos b + \cos a \sin b \\ \sin(a-b) = \sin a \cos b - \cos a \sin b \end{cases}$$

on déduit :

$$\begin{cases} \cos a \cos b = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right) \\ \sin a \sin b = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right) \\ \sin a \cos b = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right) \end{cases}$$

ainsi que : $\cos(2a) = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$

$$\boxed{\cos^2 a = \frac{1+\cos(2a)}{2}} \qquad \boxed{\sin^2 a = \frac{1-\cos(2a)}{2}}$$

Enfin, en posant $p = a + b$ et $q = a - b$ il est facile de retrouver :

$$\begin{cases} \cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2} \\ \cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2} \end{cases}$$

$$\begin{cases} \sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2} \\ \sin p - \sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2} \end{cases}$$

puisque $a = \frac{p+q}{2}$ et $b = \frac{p-q}{2}$.

Dérivées :

$$\boxed{\begin{array}{l} \cos' x = -\sin x \\ \sin' x = \cos x \end{array}}$$